CANADIAN JOURNAL OF RESEARCH

VOLUME 26 JANUARY, 1948 NUMBER 1

— SECTION A —

PHYSICAL SCIENCES

Contents

Page

Gas Dynamic Treatment of Exothermic and Endothermic Discontinuities—*D. G. Samaras* - - - - - - - - 1

A Note on an Omni-directional Array of Stacked V-Antennas—*T. P. Pepper* - - - - - - - - - - - - 22

NATIONAL RESEARCH COUNCIL
OTTAWA, CANADA

CANADIAN JOURNAL OF RESEARCH

The *Canadian Journal of Research* is issued in six sections, as follows:

A. Physical Sciences	D. Zoological Sciences
B. Chemical Sciences	E. Medical Sciences
C. Botanical Sciences	F. Technology

For the present, Sections A, C, D, and E are to be issued six times annually, and Sections B and F, twelve times annually, each section under separate cover, with separate pagination.

The *Canadian Journal of Research* is published by the National Research Council of Canada under authority of the Chairman of the Committee of the Privy Council on Scientific and Industrial Research. The *Canadian Journal of Research* is edited by a joint Editorial Board consisting of members of the National Research Council of Canada, the Royal Society of Canada, and the Chemical Institute of Canada.

Sections B and F of the *Canadian Journal of Research* have been chosen by the Chemical Institute of Canada as its medium of publication for scientific papers.

EDITORIAL BOARD

Representing National Research Council	*Representing* Royal Society of Canada	
Dr. A. R. Gordon, (*Chairman*), Head, Department of Chemistry, University of Toronto, Toronto.	Dr. A. Norman Shaw, Chairman, Department of Physics, McGill University, Montreal.	Section III
Dr. C. H. Best, The Banting and Best Department of Medical Research, University of Toronto, Toronto.	Dr. J. W. T. Spinks, Department of Chemistry, University of Saskatchewan Saskatoon.	
Dr. Robert Newton, President, University of Alberta, Edmonton.	Dr. H. S. Jackson, Head, Department of Botany, University of Toronto, Toronto.	Section V
Dr. G. H. Henderson, Professor of Mathematical Physics, Dalhousie University, Halifax.	Dr. E. Horne Craigie, Department of Zoology, University of Toronto, Toronto.	

Ex officio	*Representing* The Chemical Institute of Canada
Dr. Léo Marion, Editor-in-Chief, Division of Chemistry, National Research Laboratories, Ottawa.	Dr. R. V. V. Nicholls, Associate Professor of Chemistry, McGill University, Montreal.
Dr. H. H. Saunderson, Director, Division of Information Services, National Research Laboratories, Ottawa.	

Editorial Committee

Editor-in-Chief,	Dr. Léo Marion	Editor, Section D,	Dr. E. Horne Craigie
Editor, Section A,	Dr. A. Norman Shaw	Editor, Section E,	Dr. J. B. Collip
Editor, Section B,	Dr. J. W. T. Spinks Dr. R. V. V. Nicholls	Editor, Section F,	Dr. J. A. Anderson Dr. R. V. V. Nicholls Dr. A. Norman Shaw
Editor, Section C,	Dr. H. S. Jackson		

Manuscripts should be addressed:

Editor-in-Chief,
Canadian Journal of Research,
National Research Council, Ottawa, Canada.

Canadian Journal of Research

Issued by THE NATIONAL RESEARCH COUNCIL OF CANADA

VOL. 26, SEC. A. JANUARY, 1948 NUMBER 1

GAS DYNAMIC TREATMENT OF EXOTHERMIC AND ENDOTHERMIC DISCONTINUITIES[1]

BY D. G. SAMARAS

Abstract

Endothermic and exothermic processes in gas dynamic flows taking place in a very narrow zone may be considered as discontinuities. The variation of the static and total head density ratio, pressure ratio, and temperature ratio as well as the angle of deviation, area ratio, and exit normal Mach number have been found as functions of the entry normal Mach number and of heat addition. In addition to these, some other useful quantities such as the area ratio parameter, the difference of the square of velocities, and the normal velocity product have been evaluated. It was found that, in a discontinuity, heat can be added until the exit normal Mach number reaches unity (choking). Depending on the entry normal Mach number, only a limited amount of heat can be added at the discontinuity. An exothermic discontinuity behaves as an expansion when the entry normal Mach number is subsonic, and it is accompanied by a drop in static pressure, density, and total head pressure. An exothermic discontinuity behaves as a compression shock wave when the entry normal Mach number is supersonic, and it is accompanied by an increase in static pressure and density and a decrease in total head pressure. An endothermic discontinuity behaves always as a compression shock wave, and it is accompanied by an increase in static density, pressure, and total head pressure. It is hoped that the results and conclusions found may be useful in a better understanding of many nearly discontinuous phenomena such as flame fronts, condensation and evaporation fronts, and other similar problems.

Notation

The following symbols have been used in the analysis:

A = area,

a = local velocity of sound,

B = heat addition function,

c_p = specific heat under constant pressure,

C^* = critical velocity corresponding to total head conditions,

e = thickness of the flame front,

g = gravitational constant,

H = heat content of the combustible mixture,

[1] *Manuscript received May 5, 1947.*

Contribution from the Division of Mechanical Engineering, National Research Laboratories, Ottawa, Canada. Issued as N.R.C. No. 1670.

J = Joule's equivalent,

M = Mach number,

p = absolute pressure,

R = gas constant,

T = absolute temperature,

V = velocity,

$x = \frac{\rho_2}{\rho_1}$ = static density ratio,

$y = \frac{V_{2n}}{a_1}$ = ratio of the normal exit velocity to the inlet local velocity of sound,

$z = \frac{p_2}{\rho_1 a_1^2}$ = ratio of the exit static pressure to the momentum of the inlet velocity of sound,

γ = isentropic exponent,

δ = angle of deviation,

ν = kinematic viscosity,

ρ = absolute density,

ϕ = angle between the velocity and its normal component,

$\zeta^2 = \frac{\gamma - 1}{\gamma + 1} + \frac{2\gamma}{\gamma + 1} \cdot \frac{1}{M^2}$ = auxiliary function of the Mach number used for convenience only.

The following subscripts have been employed:

1, 2 = For the cold and hot sides of the flame front, respectively,

n = For normal,

t = For total head values (except for the velocity and Mach number denoting the tangential component).

1.0 Introduction

1.1.0 General

Since the combustion process as applied to gas turbines, propulsive ducts, and rockets is of such fundamental importance, the National Research Council of Canada has undertaken a comprehensive experimental and theoretical research program in an attempt to clarify the fundamental laws of combustion.

In the past, very little work has been done regarding the gas dynamic aspects of the combustion process in gas streams.

The first paper (7) of this series dealt with the general problem of heat addition in gas streams in one-dimensional fashion.

This is the second paper, and it deals with the general conditions pertaining in flame fronts.

Generally speaking, a flame front is defined as a thin zone where a rapid chemical reaction (combustion) takes place, accompanied by light emission.

In the case of a premixed gas, the flame zone, for reasons of simplification, called also flame surface, is defined as the surface where the burnt and unburnt gases come into contact.

In natural gas–air mixtures, the luminous zone is marked by the sharp appearance and disappearance of the C-C and C-H bands. When sodium is added to the flame, the sodium D lines appear simultaneously with these bands so that temperature measurements can be effected within the combustion zone.

The fact that the flame is accompanied by light emission is of secondary importance only; one part of the emitted light does not originate in the combustion zone, but from the still hot, just burnt, gases behind it.

The question whether the radiation emitted by a flame front is of thermal or chemical origin is of fundamental importance.

Thermal radiation is defined as that for which the intensity distribution as a function of the wave length depends only on the gas temperature and thus excludes the radiation intrinsic in the nature of the gas and the chemical luminescence (8).

It is apparent then that the radiation of a body must include not only the frequencies corresponding to the temperature radiation but also those excited by other processes. In this particular case, the additional radiation present is that due to luminescence. The energy of this radiation must be supplied from a source other than the heat energy, and, in fact, it is produced from the energy of the chemical reaction. A classical example of chemical luminescence is that of the slow oxidation of phosphorus. The reaction of phenyl magnesium iodide and chloropicrin is accompanied by the emission of green luminescence, and the reaction of formaldehyde with pyrogallol, by an orange-red luminescence (2).

According to Prettre (3), luminescence has been observed during the oxidation of practically all fuels, except hydrogen. The radiation spectrum of flames is a complicated one. It consists of the entire spectrum of solid carbon molecules, radicals of the gaseous substances, and free atoms, as well as the spectrum due to chemical luminescence.

Depending on the temperature at which combustion takes place, different types of flames may appear such as 'cool flames' and normal combustion flames.

Emeleus (3) found that the spectra of 'cool flames' are different from those of normal flames; later from the work of Kontrajew, Chou, and Henri, and Ubbelohde, it was concluded that the 'cool flames' may be attributed to the presence of formaldehyde.

The visibility of the formaldehyde luminescence is attributed to the high momentary concentration of excited formaldehyde, due to the rapid development of the reaction in the peninsula region (short induction period). At these low reaction temperatures the radiation is mostly emitted from the blue and ultraviolet, and no appreciable amount of thermal radiation is present. However, at higher temperatures, the branching is less intense and the concentrations of intermediate products build up more slowly and finally the luminescence becomes weaker.

In the combustion of ether in air, the 'cool flames' are present between 180° and 270° C. At slightly higher temperatures between 230° and 365° C. a region of 'green flames' appears. In these temperature ranges in the spectrum the C-C bands are most prevalent. The 'green flames' are very different from the bluish 'cool flames' and the white or yellow normal flames.

1.2.0 Flame Structure

It is well known that the combustion process is affected by different factors such as heat conduction (transfer of heat energy from the burnt gas to the unburnt), diffusion, especially diffusion of unburnt gas molecules into the combustion zone, and, under certain conditions, diffusion of active particles from the combustion zone into the unburnt gas.

According to the above, the flame front is only a fictitious surface. In reality, a continuous transition from the burnt to the unburnt takes place. This transition is effected in such a thin layer that, in most cases, a discontinuous variation of velocity, pressure, temperature, density, and concentration may be considered.

Little is known regarding the structure of the flame front. Lewis and von Elbe (4), making certain simplifying assumptions, succeeded in finding the order of the temperature distribution, concentration, and thickness of the combustion zone. They assumed that the sum of the chemical and of the heat energy is constant and uniformly distributed over the entire burning zone. This assumption, although not completely justified, results in a great simplification of the problem.

From these calculations, it was found that the thickness of the flame zone is of the order of 1 μ. Fig. 1 shows the experimental results (9) found by Wolfhardt with acetylene and oxygen mixtures as a function of the absolute pressure.

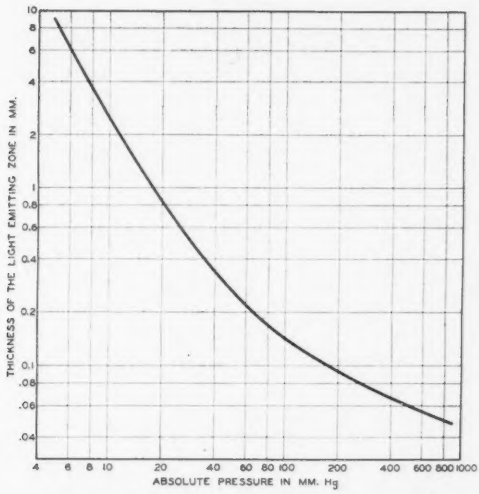

FIG. 1. *Thickness variation of the light emitting zone as a function of the absolute pressure.*

Becker (1, Chap. V) investigated the structure of a shock wave front, taking into consideration the influence of viscosity and heat transmission. His results were put in a more convenient non-dimensional form as follows:

$$\frac{e}{\nu}\sqrt{g\gamma RT_{1t}} = \frac{4}{3}\sqrt{\frac{2}{\gamma\ (\gamma + 1)}}\ \zeta\ \frac{1 + \zeta}{1 - \zeta}\,, \tag{1}$$

where

$$\zeta^2 = \frac{\gamma - 1}{\gamma + 1} + \frac{2}{\gamma + 1}\,\frac{1}{M^2}\,,$$

and were plotted in Fig. 2. This represents the intermediate special case going from the exothermic to endothermic processes.

The flame front is characterized by a very steep temperature gradient which is not so pronounced in a weak shock wave front and it is responsible for the intense radiation and heat conduction.

The flame front forms an angle with the flow lines so that the normal component of the gas velocity equals the burning velocity. Thermal expansion increases the gas velocity as well as the angle of flow lines in such a way that the flow in the stream tube remains constant. It has been observed

(5) that in slow (laminar flow) flames the refraction of a flow line begins well before it enters the luminous combustion zone, indicating a penetration of heat into the unburnt gas. In the case of turbulent flow, a less premature refraction should be expected.

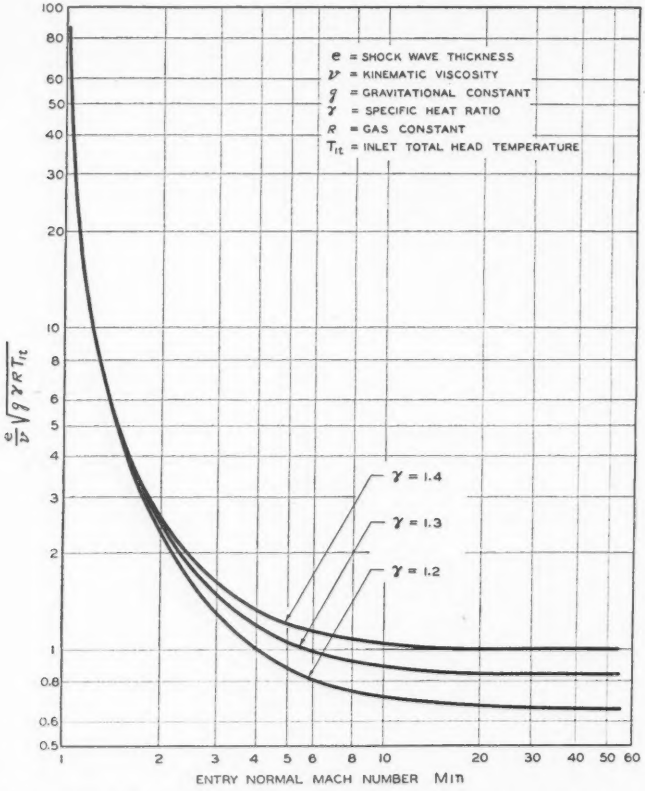
e = SHOCK WAVE THICKNESS
ν = KINEMATIC VISCOSITY
g = GRAVITATIONAL CONSTANT
γ = SPECIFIC HEAT RATIO
R = GAS CONSTANT
T_it = INLET TOTAL HEAD TEMPERATURE
γ = 1.4
γ = 1.3
γ = 1.2
ENTRY NORMAL MACH NUMBER M1n

FIG. 2. *Thickness variation of the shock wave front with the entry normal Mach number.*

In a curved flame front a large amount of vorticity is created; this results in an increase of the heat dissipation. To this effect and to the diffusion of active particles may be attributed the increased burning velocity at the tip of the Bunsen burner.

2.0 Assumptions

According to the above discussion, the flame front is assumed as a discontinuity, e.g., zero thickness through which a discontinuous change of state occurs and an increase in entropy takes place.

The influence of viscosity, heat conduction, and radiation are disregarded.

3.0 Range and Scope of Investigation

In the present treatment the endothermic and exothermic reaction fronts have been investigated.

Both subsonic and supersonic flows have been examined, including the normal combustion process as well as detonation.

In addition to the above, similar phenomena, such as condensation and evaporation fronts, are within the scope of the present investigation.

4.1.0 General Equations in the Discontinuity Front

From Fig. 3 the following equations of conservation across the discontinuity front may be written:

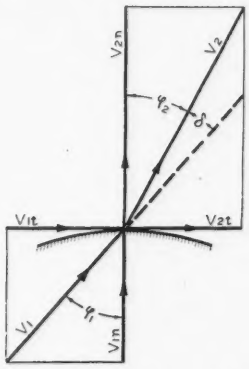

FIG. 3. *Velocity diagram of a discontinuity front element.*

The continuity

$$\rho_1 V_{1n} = \rho_2 V_{2n} \quad . \tag{2}$$

The normal component of momentum

$$p_1 + \rho_1 V_{1n}^2 = p_2 + \rho_2 V_{2n}^2 \quad . \tag{3}$$

The tangential component of momentum

$$V_{1t} = V_{2t} \quad . \tag{4}$$

The energy

$$\frac{\gamma_1}{\gamma_1 - 1} \frac{p_1}{\rho_1} + \frac{V_1^2}{2} + g J H = \frac{\gamma_2}{\gamma_2 - 1} \frac{p_2}{\rho_2} + \frac{V^2}{2} \quad . \tag{5}$$

The state equations put in the form of the sound velocity

$$a_1^2 = \gamma_1 \frac{p_1}{\rho_1} \quad , \quad a_2^2 = \gamma_2 \frac{p_2}{\rho_2} \quad . \tag{6}$$

4.2.0 The Static Density Ratio

By the use of Equations (4) and (6) we can transform Equations (2), (3), and (5) in the following manner:

$$\frac{V_{1n}}{a_1} = \frac{\rho_2}{\rho_1}\frac{V_{2n}}{a_1}, \tag{7}$$

$$\frac{1}{\gamma_1} + \frac{V_{1n}^2}{a_1^2} = \frac{p_2}{\rho_1 a_1^2} + \frac{\rho_2}{\rho_1}\frac{V_{2n}^2}{a_1^2}, \tag{8}$$

$$\frac{1}{\gamma_1 - 1} + \frac{V_{1n}^2}{2a_1^2} + g\,J\frac{H}{a_1^2} = \frac{\gamma_2}{\gamma_2 - 1}\frac{\rho_1}{\rho_2}\cdot\frac{p_2}{\rho_1 a_1^2} + \frac{V_{2n}^2}{2a_1^2}, \tag{9}$$

and calling

$$x = \frac{\rho_2}{\rho_1}, \quad y = \frac{V_{2n}}{a_1}, \quad z = \frac{p_2}{\rho_1 a_1^2},$$

$$M_{1n} = M_1\cos\phi_1 = \frac{V_{1n}}{a_1}, \quad M_{2n} = M_2\cos(\phi_1 - \delta) = \frac{V_{2n}}{a_2},$$

$$B = g\,J\frac{H}{a_1^2} = \frac{c_{p_{12}}}{c_{p_1}}\left(\frac{T_{2t}}{T_{1t}} - 1\right)\left(\frac{1}{\gamma_1 - 1} + \frac{M_1^2}{2}\right),$$

we obtain the following equations:

$$xy = M_{1n} \tag{7'}$$

$$\frac{1}{\gamma_1} + M_{1n}^2 = z + x\,y^2 \tag{8'}$$

$$\frac{1}{\gamma_1 - 1} + \frac{M_{1n}^2}{2} + B = \frac{\gamma_2}{\gamma_2 - 1}\frac{z}{x} + \frac{y^2}{2}. \tag{9'}$$

After some algebraic transformations we finally obtain

$$M_{1n}^2(1 + \gamma_2)\left(\frac{1}{x}\right)^2 - 2\,\gamma_2\left(\frac{1}{\gamma_1} + M_{1n}^2\right)\left(\frac{1}{x}\right) + (\gamma_2 - 1)\left[\frac{2}{\gamma_1 - 1} + M_{1n}^2 + 2\,B\right] = 0. \tag{10}$$

The solution of this equation is as follows

$$\frac{\rho_1}{\rho_2} = \frac{1}{x} = \frac{\left(\frac{\gamma_2}{\gamma_1} + \gamma_2 M_{1n}^2\right) \pm \sqrt{\left(\frac{\gamma_2}{\gamma_1} + \gamma_2 M_{1n}^2\right)^2 - M_{1n}^2(\gamma_2^2 - 1)\left[\frac{2}{\gamma_1 - 1} + M_{1n}^2 + 2B\right]}}{(\gamma_2 + 1)\,M_{1n}^2}. \tag{11}$$

The static density curves are asymptotic to two density ratio lines

$$\frac{\rho_1}{\rho_2} = 1.0 \quad \text{and} \quad \frac{\rho_1}{\rho_2} = \frac{\gamma_2 - 1}{\gamma_2 + 1},$$

as is clearly seen from Fig. 4.

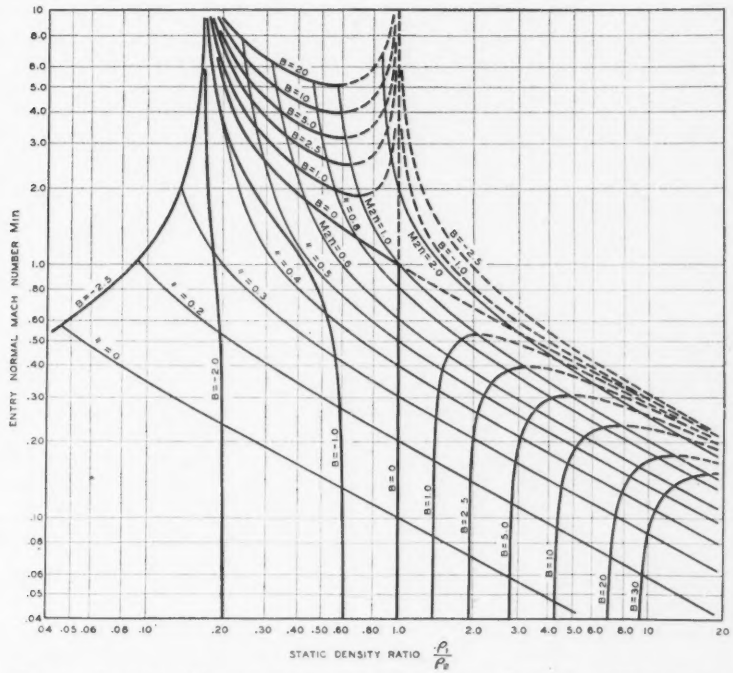

FIG. 4. *Variation of the static density ratio with the entry and exit normal Mach numbers.*

4.3.0 THE STATIC PRESSURE RATIO

The static pressure ratio across the discontinuity front may be found as follows

$$\frac{p_2}{p_1} = \frac{p_2}{\rho_1 a_1^2} \frac{\rho_1 a_1^2}{p_1} = \gamma_1 z \ , \tag{12}$$

and taking into consideration Equations (7′) and (8′) we obtain finally

$$\frac{p_2}{p_1} = 1 + \gamma_1 M_{1n}^2 \left(1 - \frac{\rho_1}{\rho_2}\right). \tag{13}$$

4.4.0 THE STATIC TEMPERATURE RATIO

This is found to be as follows:

$$\frac{T_2}{T_1} = \frac{R_1}{R_2}\left[1 + \gamma_1 M_{1n}^2 \left(1 - \frac{\rho_1}{\rho_2}\right)\right] \frac{\rho_1}{\rho_2}. \tag{14}$$

4.5.0 The Angle of Deviation

This may be found from the following equation:

$$\frac{tg\,(\phi_1 - \delta)}{tg\,\phi_1} = \frac{\rho_2}{\rho_1}\,. \tag{15}$$

4.6.0 The Mach Number Ratio

After a few simple algebraic calculations the normal Mach number ratio is found to be

$$\frac{M_{2n}^2}{M_{1n}^2} = \frac{M_2^2 \cos^2(\phi_1 - \delta)}{M_1^2 \cos^2\phi_1} = \frac{\dfrac{\gamma_1}{\gamma_2}\cdot\dfrac{\rho_1}{\rho_2}}{1 + \gamma_1 M_{1n}^2\left(1 - \dfrac{\rho_1}{\rho_2}\right)}\,. \tag{16}$$

4.7.0. The Total Head Pressure Ratio

After some simple algebraic transformations the total head pressure ratio was found to be:

$$\frac{p_{2t}}{p_{1t}} = \frac{\left[1 + \gamma_1 M_{1n}^2\left(1 - \dfrac{\rho_1}{\rho_2}\right)\right]}{\left[1 + \dfrac{\gamma_1 - 1}{2}\dfrac{M_{1n}^2}{\cos^2\phi_1}\right]^{\frac{\gamma_1}{\gamma_1 - 1}}}\cdot\left\{1 + \frac{\gamma_2 - 1}{2\gamma_2}\left[\frac{\gamma_1 M_{1n}^2\left(\dfrac{\rho_1}{\rho_2} + \dfrac{\rho_2}{\rho_1}\,tg^2\phi_1\right)}{1 + \gamma_1 M_{1n}^2\left(1 - \dfrac{\rho_1}{\rho_2}\right)}\right]\right\}. \tag{17}$$

In the case of a normal front $\phi = 0$ the total head pressure ratio reaches its minimum value.

4.8.0. The Normal Velocity Ratio

From Equation (2) it is seen that the normal velocity ratio is as follows:

$$\frac{V_{2n}}{V_{1n}} = \frac{\rho_1}{\rho_2}\,. \tag{18}$$

4.9.0 The Normal Area Ratio

From Equations (2), (16), and (18) the area ratio may be found

$$\frac{A_{2n}}{A_{1n}}\cdot\frac{M_2}{M_1} = \sqrt{\frac{\dfrac{\gamma_1}{\gamma_2}\dfrac{\rho_1}{\rho_2}}{1 + \gamma_1 M_{1n}^2\left(1 - \dfrac{\rho_1}{\rho_2}\right)}}\,. \tag{19}$$

This parameter is plotted against M_{1n} in Fig. 5.

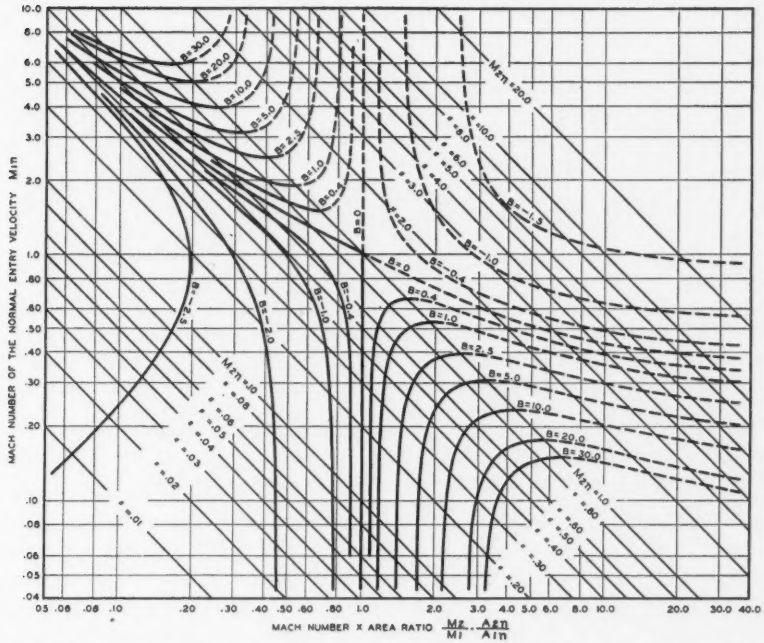

FIG. 5. *Variation of the heat addition parameter with the entry and exit normal Mach numbers in a discontinuity front.*

4.10.0 The Difference of the Square of Velocities

After some simple algebraic calculations, the difference of the square of velocities was found to be:

$$\frac{V_2^2 - V_1^2}{a_1^2} = M_{1n}^2 \left[\left(\frac{\rho_1}{\rho_2} \right)^2 - 1 \right] . \tag{20}$$

4.11.0 The Normal Velocity Product

In the Appendix the product of the normal velocities has been derived and is as follows:

$$\frac{V_{1n}\, V_{2n}}{C_1^{*2}} = \left(\frac{\gamma_1 + 1}{\gamma_2 + 1} \right) \left\{ \left[\frac{\gamma_2}{\gamma_1} + \frac{\gamma_1 - 1}{2} \frac{M_{1t}^2}{1 + \frac{\gamma_1 - 1}{2} M_1^2} \right] - \frac{\frac{\gamma_2}{\gamma_1}}{\frac{\rho_1}{\rho_2} - 1} \left[\left(\frac{T_{2t}}{T_{1t}} - 1 \right) + \frac{\rho_1}{\rho_2} \cdot \frac{\gamma_1 - \gamma_2}{2\,\gamma_2} \frac{M_1^2}{1 + \frac{\gamma_1 - 1}{2} M_1^2} \right] \right\} . \tag{21}$$

Equation (21) can also be written in another form as follows:

$$\frac{V_{1n}\,V_{2n}}{C_1^{*2}} = \left(\frac{\gamma_1+1}{\gamma_2+1}\right)\left\{\frac{\gamma_2}{\gamma_1}\left(\frac{\dfrac{\rho_1}{\rho_2}-\dfrac{T_{2t}}{T_{1t}}}{\dfrac{\rho_1}{\rho_2}-1}\right)\right.$$

$$\left. + \frac{\dfrac{\gamma_1-1}{2}M_1^2}{1+\dfrac{\gamma_1-1}{2}M_1^2}\left[\frac{M_{1t}^2}{M_1^2} - \frac{\rho_1}{\rho_1-\rho_2}\left(\frac{\gamma_1-\gamma_2}{\gamma_1(\gamma_1-1)}\right)\right]\right\}. \qquad (22)$$

5.0 Results and Discussion

Figs. 4 and 5 show the variation of the static density and area ratio parameter with the entry and exit normal Mach numbers for different values of the heat addition function B.

It is seen that the exit normal Mach number has unity as a limiting value. For a given subsonic normal entry Mach number through an exothermic discontinuity, a maximum heat addition function appears, corresponding to unity exit normal Mach number, and the maximum entry normal Mach number decreases with increasing heat addition function.

In a given supersonic normal entry velocity through an exothermic discontinuity, a minimum heat addition function appears, corresponding to unity exit normal Mach number, and the minimum entry normal Mach number increases with increasing heat addition function.

In endothermic discontinuities, no extreme values of the entry normal Mach number appear.

For supersonic normal entry velocities passing through exothermic discontinuities, the resulting exit static density is larger than the entry one and the ratio of the entry to the exit static density decreases with increasing heat addition function; consequently the discontinuity behaves like a compression shock wave. For a subsonic entry normal velocity the reverse is true and an acceleration is realized through the discontinuity. In each case, the static density ratio decreases with increasing entry normal Mach number. When the discontinuity is endothermic the exit static density is always larger than the entry and the static density ratio decreases with increasing heat addition function.

Figs. 6 and 7 show the variation of the entry and exit normal Mach numbers with the minimum total head pressure ratio and the heat addition function.

It is seen that when the flow passes through an exothermic discontinuity, the total head pressure ratio is smaller than unity (loss of momentum) and the reverse is true when it passes through an endothermic discontinuity (Mach number slightly greater than one).

In the endothermic case, a maximum value of the total head pressure ratio $\left(\frac{p_{2t}}{p_{1t}}\right)_{min}$ appears at an entry normal Mach number slightly smaller than unity, tending towards unity as the heat addition function tends to zero and accompanied by an increase of the exit normal Mach number. The maximum value of the total head pressure ratio $\left(\frac{p_{2t}}{p_{1t}}\right)_{min} = 1.235$. When the flow passes

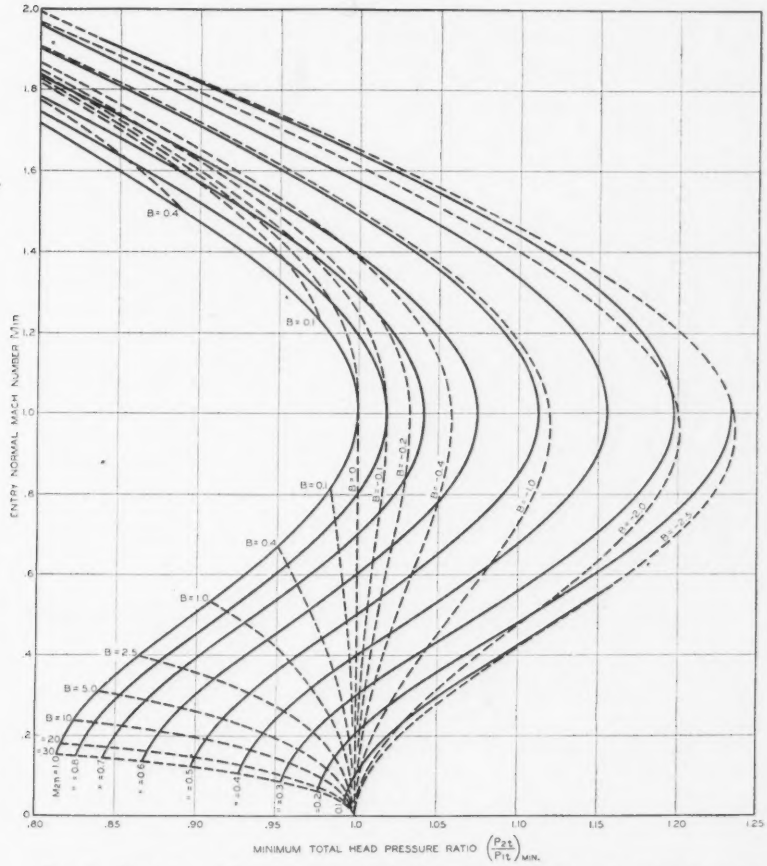

FIG. 6. *Variation of the minimum total head pressure with the entry and exit normal Mach numbers.*

through an exothermic discontinuity and the entry normal Mach number is smaller than unity, the total head pressure ratio is always $\left(\frac{p_{2t}}{p_{1t}}\right)_{min} > 0.8$, and increases with an increasing entry normal Mach number and the heat addition function. In the case when the entry normal Mach number is larger than

unity, no lower limit of the total head pressure ratio exists and it decreases with increasing entry normal Mach number and heat addition function B.

From Fig. 7 it is seen that for a given total head pressure ratio $\left(\frac{p_{2t}}{p_{1t}}\right)_{min}$ in the exothermic case, a maximum exit normal Mach number appears.

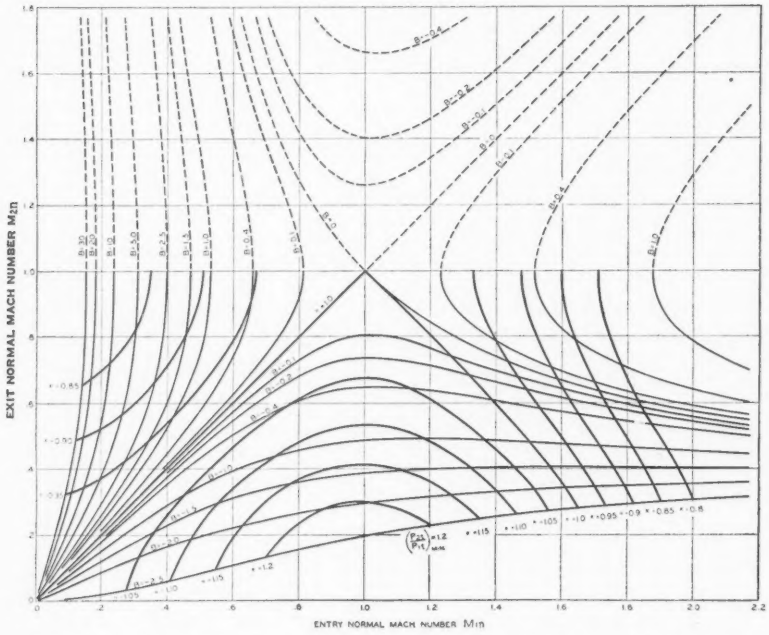

FIG. 7. *Variation of the exit normal Mach number with the entry normal Mach number.*

In Fig. 8 is shown the variation of the static pressure difference ratio across the discontinuity with the entry and exit normal Mach number and the heat addition function. It is seen that when the entry normal Mach number is smaller than unity, a drop in static pressure occurs on passing through the exothermic discontinuity, and the larger this pressure drop, the larger the entry normal Mach number and the larger the heat addition function.

If the entry normal Mach number is larger than unity, an increase in static pressure occurs and this increase is larger, the larger the entry normal Mach number and the smaller the heat addition function.

Figs. 9, 10, 11, 12, and 13 show the variation of the angle of deviation with the angle of incidence and the entry Mach number for different values of the heat addition function $B = -2.5, -1.0, 0, 2.5$, and 10, respectively.

It is seen that the maximum (negative) deviation for infinite entry Mach number, $M_1 \rightarrow \infty$, varies little with the heat addition function and is slightly

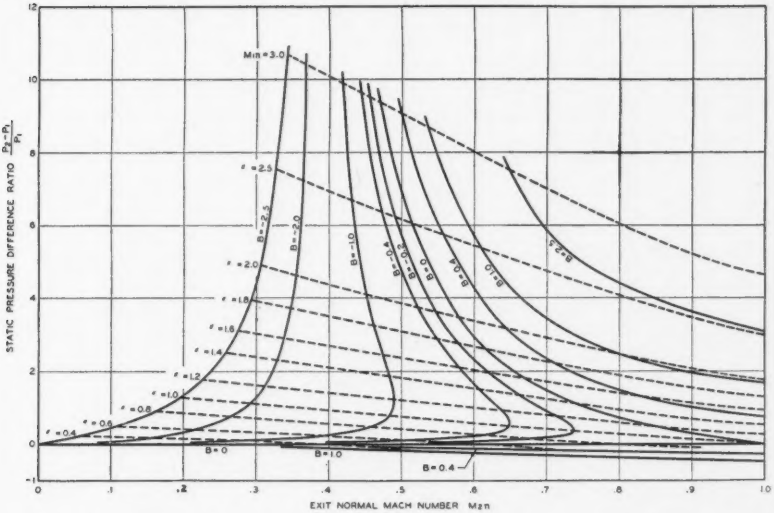

FIG. 8. *Variation of the static pressure difference with the entry and exit normal Mach numbers.*

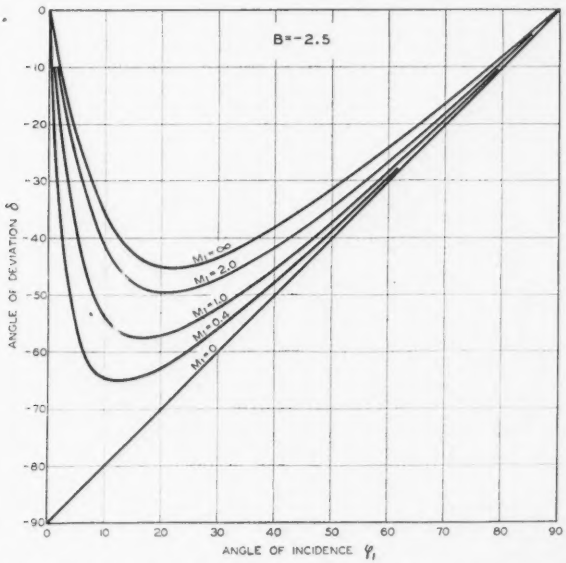

FIG. 9. *Variation of the angle of deviation with the angle of incidence and the entry Mach number.*

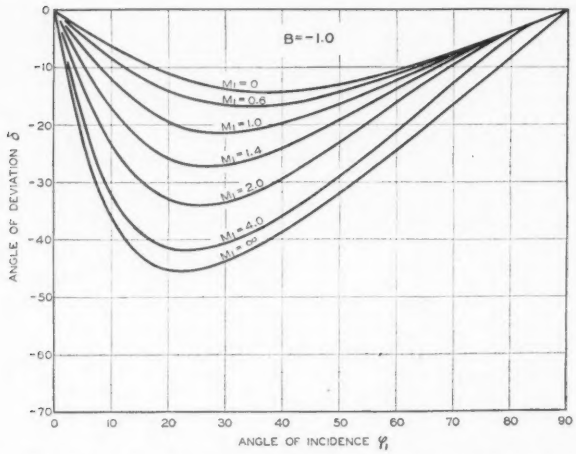

FIG. 10. *Variation of the angle of deviation with the angle of incidence and the entry Mach number.*

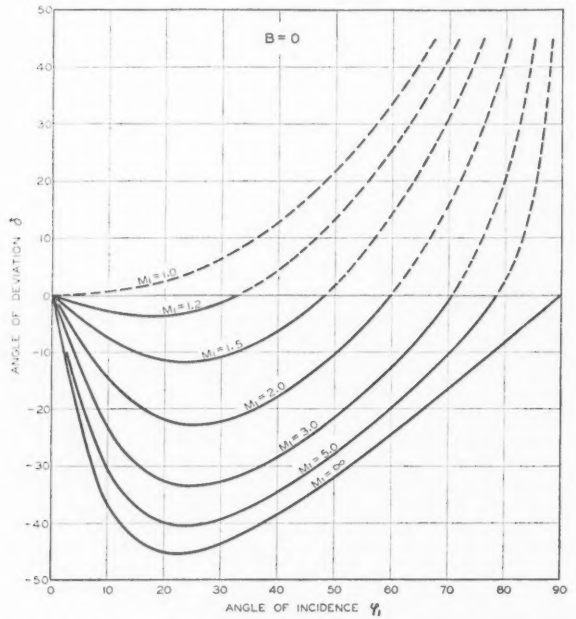

FIG. 11. *Variation of the angle of deviation with the angle of incidence and the entry Mach number.*

larger than $-45°$. Again the maximum angle of deviation for zero entry Mach number, $M_1 \rightarrow 0$, increases with the heat addition parameter from $\delta = -90°$, $B = 2.5$ to $\delta = 90°$, $B \rightarrow \infty$.

For all endothermic cases, including the isoenergetic, for a given entry Mach number a maximum deviation appears and its value decreases with increasing heat addition function.

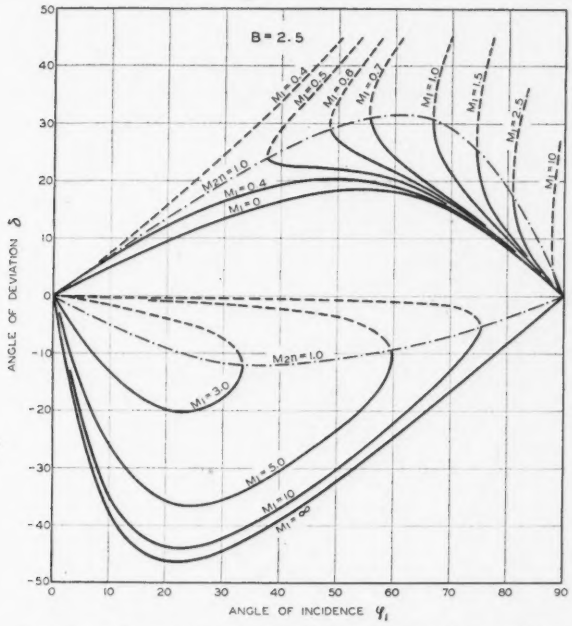

FIG. 12. *Variation of the angle of deviation with the angle of incidence and the entry Mach number.*

In exothermic discontinuities and for low entry Mach numbers (depending on the value of the heat addition parameter), maximum deviation angles appear, until a certain value of the entry Mach number is reached. This is changed into an inflection point and the deviation curve reaches a limiting value when the exit normal Mach number reaches unity. The larger the heat addition function the smaller the M_1 value at which the inflection appears.

For supersonic inlet flows and small angle of incidence the deviation angle is negative, reaching a maximum (negative) value after ending in a limiting line where the exit normal Mach number is unity. Above this angle of incidence no solution exists until another limiting value corresponding to an

exit normal Mach number of unity is reached. Beyond this limiting value a positive angle of deviation appears and its value decreases with increasing angle of incidence.

From the above discussion it may be concluded that, by passing through the discontinuity when the angle of deviation is negative, a compression shock wave occurs, and when the deviation is positive, an expansion occurs.

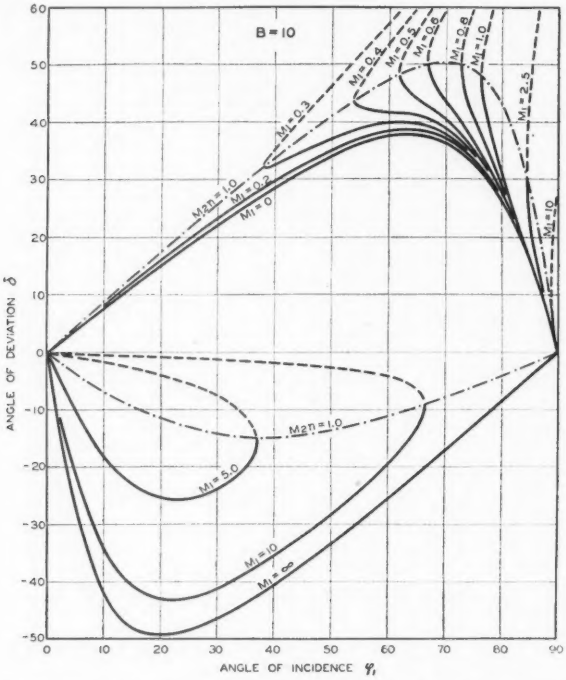

FIG. 13. *Variation of the angle of deviation with the angle of incidence and the entry Mach number.*

The velocity product is not a constant as in the cases examined by L. Prandtl and Th. Meyer (6) but a complicated function of the Mach numbers, densities, and temperatures.

6.0 Conclusions

From the above analysis and discussion, the following conclusions are drawn:

The maximum exit normal Mach number through a discontinuity front is unity.

In an exothermic discontinuity, depending on the value of the heat addition function, a maximum or a minimum value of the entry normal Mach number appears corresponding to the subsonic or supersonic normal flow. In an exothermic discontinuity, when the entry normal Mach number is less than unity, the discontinuity behaves like an expansion and is accompanied by a drop in static pressure, density, and total head pressure. In the reverse case, of a normal Mach number larger than unity, it behaves as a compression shock wave and is accompanied by an increase in static pressure and density and a decrease in total head pressure.

When the flow passes through an endothermic discontinuity, the discontinuity behaves as a compression shock wave, and is accompanied by an increase in static density, pressure, and total head pressure.

References

1. ACKERET, J. *In* Handbuch der Physik. Vol. 7. Julius Springer, Berlin. 1927.
2. AUDUBERT, R. Angew. Chem. 51 : 153. 1938.
3. JOST, W. Explosion and combustion processes in gases. *Translated from the German by* Huber O. Croft. McGraw-Hill Book Co., Inc., New York. 1946.
4. LEWIS, B. and VON ELBE, G. Combustion, flames and explosions of gases. Cambridge Univ. Press, London. 1938.
5. LEWIS, B. and VON ELBE, G. J. Chem. Phys. 11 : 75. 1943.
6. MEYER, T. VDI Forschungsheft, 62 : 31. 1908.
7. SAMARAS, D. G. Can. J. Research, F, 24 : 272. 1946.
8. WERNEBURG, J. Forsch. Gebiete Ingenieurw. 10 : 61. 1939.
9. WOLFHARDT, H. G. Z. tech. Physik, 24 : 206. 1943.

APPENDIX

Derivation of the Normal Velocity Product

The energy equations on either side of the discontinuity front may be written as follows

$$\left.\begin{aligned} \frac{p_1}{\rho_1} &= \frac{p_{1t}}{\rho_{1t}} - \frac{\gamma_1 - 1}{\gamma_1}\frac{V_1^2}{2} \\ \frac{p_2}{\rho_2} &= \frac{p_{2t}}{\rho_{2t}} - \frac{\gamma_2 - 1}{\gamma_2}\frac{V_2^2}{2} = \frac{p_{1t}}{\rho_{1t}} + \left(\frac{p_{2t}}{\rho_{2t}} - \frac{p_{1t}}{\rho_{1t}}\right) - \frac{\gamma_2 - 1}{\gamma_2}\cdot\frac{V_2^2}{2} \end{aligned}\right\}, \quad (23)$$

and after some transformation the following may be obtained

$$p_2 - p_1 = (\rho_2 - \rho_1)\frac{p_{1t}}{\rho_{1t}} + \rho_2\left(\frac{p_{2t}}{\rho_{2t}} - \frac{p_{1t}}{\rho_{1t}}\right) - \frac{1}{2}\left(\frac{\gamma_2 - 1}{\gamma_2}\rho_2 V_2^2 - \frac{\gamma_1 - 1}{\gamma_1}\rho_1 V_1^2\right). \quad (24)$$

By using the momentum Equations (3) and (4) the following is found:

$$p_2 - p_1 = -(\rho_2 - \rho_1)\, V_i^2 + \rho_1 V_1^2 - \rho_2 V_2^2, \quad (25)$$

and from Equations (24) and (25)

$$\frac{\gamma_2+1}{2\gamma_2}(p_2-p_1)=(\rho_2-\rho_1)\left[\frac{p_{1t}}{\rho_{1t}}+\frac{\gamma_2-1}{2\gamma_2}V_t^2\right]$$

$$+\rho_2\left(\frac{p_{2t}}{\rho_{2t}}-\frac{p_{1t}}{\rho_{1t}}\right)+\frac{\rho_1 V_1^2}{2}\left(\frac{\gamma_1-1}{\gamma_1}-\frac{\gamma_2-1}{\gamma_2}\right), \qquad (26)$$

but from Equations (3) and (18) the following may be obtained

$$p_2-p_1=\rho_1 V_{1n}^2-\rho_2 V_{2n}^2=\rho_1^2 V_{1n}^2\left(\frac{1}{\rho_1}-\frac{1}{\rho_2}\right)=\rho_1^2 V_{1n}^2\left(\frac{\rho_2-\rho_1}{\rho_1\rho_2}\right),$$

and finally

$$V_{1n}V_{2n}=\frac{p_2-p_1}{\rho_2-\rho_1}. \qquad (27)$$

From Equations (26) and (27) the following may be found

$$V_{1n}V_{2n}=\frac{2\gamma_2}{\gamma_2+1}\left\{\left(\frac{p_{1t}}{\rho_{1t}}+\frac{\gamma_2-1}{2\gamma_2}V_{1t}^2\right)+\frac{\rho_2}{\rho_2-\rho_1}\left[\left(\frac{p_{2t}}{\rho_{2t}}-\frac{p_{1t}}{\rho_{1t}}\right)\right.\right.$$

$$\left.\left.+\frac{1}{2}\frac{\rho_1}{\rho_2}V_1^2\left(\frac{\gamma_1-1}{\gamma_1}-\frac{\gamma_2-1}{\gamma_2}\right)\right]\right\}. \qquad (28)$$

The critical velocity corresponding to total head conditions is given by:

$$C_1^{*2}=\frac{2\gamma_1}{\gamma_1+1}\cdot\frac{p_{1t}}{\rho_{1t}}=\frac{2\gamma_1}{\gamma_1+1}gRT_{1t}, \qquad (29)$$

and substituting into Equation (28) after some simple transformations, we finally obtain:

$$\frac{V_{1n}V_{2n}}{C_1^{*2}}=\left(\frac{\gamma_1+1}{\gamma_2+1}\right)\left\{\left[\frac{\gamma_2}{\gamma_1}+\frac{\gamma_1-1}{2}\frac{M_{1t}^2}{1+\frac{\gamma_1-1}{2}M_1^2}\right]\right.$$

$$\left.-\frac{\frac{\gamma_2}{\gamma_1}}{\frac{\rho_1}{\rho_2}-1}\left[\left(\frac{T_{2t}}{T_{1t}}-1\right)+\frac{\rho_1}{\rho_2}\frac{\gamma_1-\gamma_2}{2\gamma_2}\frac{M_1^2}{1+\frac{\gamma_1-1}{2}M_1^2}\right]\right\}, \qquad (21)$$

and by rearranging this equation after some transformation we finally obtain:

$$\frac{V_{1n}V_{2n}}{C_1^{*2}} = \left(\frac{\gamma_1+1}{\gamma_2+1}\right)\left\{\frac{\gamma_2}{\gamma_1}\left(\frac{\dfrac{\rho_1}{\rho_2}-\dfrac{T_{2t}}{T_{1t}}}{\dfrac{\rho_1}{\rho_2}-1}\right)\right.$$

$$\left. + \frac{\dfrac{\gamma_1-1}{2}M_1^2}{1+\dfrac{\gamma_1-1}{2}M_1^2}\left[\frac{M_{1t}^2}{M_1^2} - \frac{\rho_1}{\rho_1-\rho_2}\left(\frac{\gamma_1-\gamma_2}{\gamma_1(\gamma_1-1)}\right)\right]\right\}. \qquad (22)$$

A NOTE ON AN OMNI-DIRECTIONAL ARRAY OF STACKED V-ANTENNAS[1]

By T. P. Pepper

Abstract

An omni-directional antenna array, which might find use in the frequency-modulation broadcast field, is described.

An omni-directional antenna-array, which might find use in the frequency-modulation broadcast field, was developed during the war by the National Research Council of Canada, as an omni-directional radar beacon for the Royal Canadian Air Force (see photograph). Five simple V-antennas, each consisting of two mutually perpendicular end-fed half-wave dipoles, were stacked at half-wave spacing to give the required gain. Each V-element was fed from a double coaxial transmission line, which in turn was center-fed from balanced BA4 (RG22/U) cable.

The two dipoles of each V were mounted, one on each side of the double coaxial feed, and set to make an angle of approximately 45° with the center line through the feed, giving a total of approximately 90° between dipoles. The position of the dipoles was interchanged for alternate V's, to give the required phasing.

A reasonably circular pattern, and a power gain quite comparable with other five-bay stacks proposed for frequency-modulation broadcast purposes, was obtained. At the same time, matching problems proved to be very much simpler than for stacks of some of the so-called loop antennas.

The entire array was weatherproofed, and found simple enough for mounting on a wooden frame.

[1] *Manuscript received June 3, 1947.*

Contribution from the Electrical Engineering and Radio Branch, National Research Laboratories, Ottawa, Canada. Issued as N.R.C. No. 1656.

PLATE I

The experimental model of a stacked V-antenna.

CANADIAN JOURNAL OF RESEARCH

Notes on the Preparation of Copy

GENERAL:—Manuscripts should be typewritten, double spaced, and the **original and one extra copy** submitted. Style, arrangement, spelling, and abbreviations should conform to the usage of this Journal. Names of all simple compounds, rather than their formulae, should be used in the text. Greek letters or unusual signs should be written plainly or explained by marginal notes. Superscripts and subscripts must be legible and carefully placed. Manuscripts should be carefully checked before being submitted, to reduce the need for changes after the type has been set. If authors require changes to be made after the type is set, they will be charged for changes that are considered to be excessive. **All pages, whether text, figures, or tables, should be numbered.**

ABSTRACT:—**An abstract of not more than about 200 words, indicating the scope of the work and the principal findings, is required.**

ILLUSTRATIONS:

(i) *Line Drawings:*—All lines should be of sufficient thickness to reproduce well. Drawings should be carefully made with India ink on white drawing paper, blue tracing linen, or co-ordinate paper **ruled in blue only**; any co-ordinate lines that are to appear in the reproduction should be ruled in black ink. Paper ruled in **green, yellow, or red should not be used** unless it is desired to have all the co-ordinate lines show. Lettering and numerals should be neatly done in India ink preferably with a stencil (**do not use typewriting**) and be of such size that they will be legible and not less than one millimeter in height when reproduced in a cut three inches wide. All experimental points should be carefully drawn with instruments. Illustrations need not be more than two or three times the size of the desired reproduction, but the ratio of height to width should conform with that of the type page. **The original drawings and one set of small but clear photographic copies are to be submitted.**

(ii) *Photographs:*—Prints should be made on glossy paper, with strong contrasts; they should be trimmed to remove all extraneous material so that essential features only are shown. Photographs should be submitted **in duplicate**; if they are to be reproduced in groups, one set should be so arranged and mounted on cardboard with rubber cement; the duplicate set should be unmounted.

(iii) *General:*—**The author's name, title of paper, and figure number should be written in the lower left-hand corner (outside the illustration proper) of the sheets on which the illustrations appear.** Captions should not be written on the illustrations, but typed on a separate page of the manuscript. All figures (including each figure of the plates) should be numbered consecutively from 1 up (arabic numerals). **Each figure should be referred to in the text.** If authors desire to alter a cut, they will be charged for the new cut.

TABLES:—Titles should be given for all tables, which should be numbered in Roman numerals. Column heads should be brief and textual matter in tables confined to a minimum. **Each table should be referred to in the text.**

REFERENCES:—**These should be listed alphabetically by authors' names, numbered in that order, and placed at the end of the paper.** The form of literature citation should be that used in the respective sections of this Journal. **Titles of papers should not be given in references listed in Sections A, B, E, and F, but must be given in references listed in Sections C and D.** The first page only of the references cited in papers appearing in Sections A, B, and E should be given. **All citations should be checked with the original articles.** Each citation should be referred to in the text by means of the key number; in Sections C and D the author's name and the date of publication may be included with the key number if desired.

The *Canadian Journal of Research* conforms in general with the practice outlined in the *Canadian Government Editorial Style Manual*, published by the Department of Public Printing and Stationery, Ottawa.

Reprints

Fifty reprints of each paper without covers are supplied free. Additional reprints, if required, will be supplied according to a prescribed schedule of charges. On request, covers can be furnished at cost.